# Orbit Determination of VSOP-2 with GPS Measurements

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## **BIOGRAPHIES**

Sien-Chong Wu received his Ph.D. degree in Electrical Engineering in 1973 from the University of Waterloo, Canada. He is currently a Principal Engineer in the Tracking Systems and Applications Section at JPL. He has been involved with the development of various tracking systems for deep-space as well as near-Earth space vehicles, and their applications to precision geodesy. His current interest is in the applications of GPS technology for precise positioning in space.

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# **ABSTRACT**

VSOP-2 is a space VLBI mission in a highly elliptic orbit. To perform its science mission, VSOP-2 requires centimeter level orbit determination. In this paper we investigate the ability to satisfy these extreme orbit determination requirement using GPS data with several augmentations. High precision accelerometers are used for connecting orbit solutions at different epochs, especially near the apogee where GPS measurements are scarce. Additionally, GPS-like measurements transmitted by VSOP-2 and acquired by a network further improve the orbit determination accuracy near apogee. A covariance/ simulation analysis is carried out to assess the potential orbit determination accuracy with different data assumptions and combinations. The results of this analysis indicates that orbit position at lower altitudes can be determined to sub-cm accuracy with conventional GPS data and moderate quality accelerometry. At higher altitudes, the scarcity of conventional GPS data can be

strengthened with an ultra-precise accelerometry and the addition of GPS-like data from 12 ground stations. With proper data combination and filtering process, the centimeter-level accuracy can be expected at all altitudes.

#### INTRODUCTION

VSOP-2 is a Japanese VLBI (Very-Long Baseline Interferometry) Space Observatory Program follow-on flight project. The spacecraft will be put into a highly elliptical spacecraft orbit in the latter half of this decade. Astronomical observations will be taken in the form of space-based VLBI with a precision well exceeding that can be obtained with ground-based VLBI. Precise determination of VSOP-2 orbit to centimeter accuracy at all altitudes is highly desirable to fully exploit the value of the scientific observations. This level of orbit accuracy cannot rely on conventional dynamic orbit determination techniques due to modeling uncertainties of the orbit dynamics.

For precise VSOP-2 orbit determination, onboard GPS measurements are considered. Such measurements have demonstrated a few-cm accuracy for the determination of low Earth orbits [1,2]. VSOP-2 will have a highly elliptical orbit with an apogee altitude of about 30,000 km and perigee of about 1000 km. At lower altitudes near the perigee, numerous GPS satellites are visible and a good orbit solution can be expected. However, at higher altitudes, very few GPS satellites at a time are visible, even with an all-in-view onboard antenna configuration; furthermore, GPS signals are very week for altitudes above ~20,000 km. To strengthen the orbit determination, high-quality accelerometry can be used to supplement GPS measurements. Accelerometry will bridge orbit positions and velocities over periods of time when GPS measurements are unable to provide good solutions.

A covariance analysis is carried out to assess the potential VSOP-2 orbit determination accuracy. The results of this analysis show an accuracy well below 1 cm in all three components near the perigee. At higher altitudes where GPS measurements are scarce, the orbit error grows to

about 10 cm. The poorer orbit accuracy at higher altitudes can be improved by adding GPS-like measurements. With this technique, GPS-like signals are transmitted from VSOP-2 and received by ground stations as if the signals were transmitted from yet another GPS. Since such signals are most needed at high altitudes, the transmitting antenna can have a narrow beam and thus high gain for reduced data noise. The orbit error near apogee can be reduced by a factor of 2 to 5 depending on the number of ground tracking sites involved.

## **VSOP-2 ORBIT AND TRACKING DATA**

VSOP-2 will have a perigee altitude of  $\sim 1000$  km and an apogee altitude of  $\sim 30,000$  km, resulting in an orbit period of  $\sim 9$  hours. The nominal inclination is 31°. Fig. 1 shows the altitude of a VSOP-2 orbit as a function of time over 2 orbit periods.

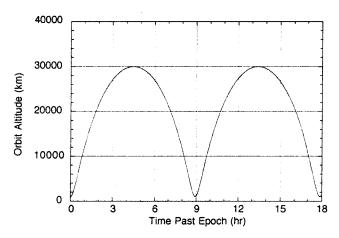


Fig. 1. VSOP-2 altitude as a fundtion of time

The major tracking data for the orbit determination of VSOP-2 are GPS pseudorange and carrier phases. The altitude of the GPS satellites is ~20,500 km. The beamwidth of the GPS transmitting antenna is designed to illuminate near-earth users. The half-width of the mainbeams are 22° and 27° respectively at L1 and L2 frequencies [3]. When the user altitude is greater than ~3,500 km, it will be outside of the mainbeam illumination of a given GPS satellite for a fraction of the time. Eventually, when the user altitude is greater than the GPS altitude, there will be no GPS illumination from above, as shown in Fig. 2. A side looking or a downward looking antenna will be required to obtain any GPS observations.

At altitudes near the VSOP-2 apogee only 0 to 3 GPS can be observed at any given time, even with an antenna system covering all directions. In general, the GPS signal will be very week for altitudes above 20,000 km. To strengthen the orbit determination, high-quality accelerometers can be used together with GPS measurements. Accelerometry will connects orbit

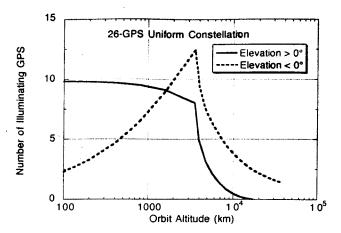


Fig. 2. GPS visibility above and below VSOP-2 local horizon

positions and velocities over periods of time when GPS measurements are unable to provide good solutions.

The orbit accuracy at higher altitudes can also be improved by adding GPS-like measurements [4]. Such measurements are derived from signal transmitted by VSOP-2 and received by ground stations the same way as receiving GPS signals. Since such signals are most needed at high altitudes, the transmitting antenna can have a narrow beam and thus high gain for better data quality.

## **COVARIANCE ANALYSIS**

To assess the potential VSOP-2 orbit determination accuracy, a covariance analysis is carried out. A constellation of 26 GPS satellites is assumed. Pseudorange and carrier phase measurements are simulated between VSOP-2 and all illuminating GPS at a 5-minute interval over 18 hours, or 2 revolutions of VSOP-2 orbit. GPS-like pseudoranges and carrier phases from 6 ground stations are simulated when VSOP-2 altitude is higher than 20,000 km. 3-D accelerometry is also simulated at the 5-minute interval. The data quality assumed is as shown in Table 1:

**Table 1. Tracking Data Quality** 

Data Sampling Time: 5 minutes

GPS Pseudorange: 10 cm

GPS Carrier Phase: 1 mm

GPS-like Pseudorange: 10 cm

GPS-like Carrier Phase: 5 mm

Accelerometry: 1 nm/sec<sup>2</sup>

Orbit dynamics rely on accelerometry and the oblateness model of the Earth. The later can be modeled with high accuracy and the effects of its mismodeling error will be ignored here. In this analysis, the errors due to GPS orbits and clocks, which will be determined to high accuracy by a network of ground observing stations, are simulated by the difference between JPL's rapid GPS orbit solutions and JPL's precise GPS orbit solutions [5]. These differences have an RMS value of ~20 cm. Their effects are scaled down by a factor of 4 to reflect the expected GPS orbit and clock errors.

# **Conventional GPS and Accelerometry**

Fig. 3 shows the elevation angles of the observed GPS satellites as a function of time. At low altitudes near the perigee (0, 9 and 18 hours past epoch), numerous GPS satellites are visible both above and below the local horizon. At high altitude near the apogee (4.5 and 13.5 hours past epoch), no GPS is visible from above and only 0 to 3 GPS are visible from below.

The results of the covariance analysis with conventional GPS and accelerometry are summarized in Figs. 4 where the formal error of VSOP-2 orbit determination is shown as a function of time over 2 orbital periods.

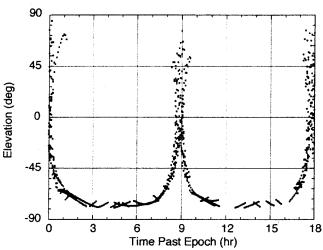


Fig. 3. Elevation angle of visible GPS from VSOP-2 (measured from the local horizon at VSOP-2, positive toward local zenith)

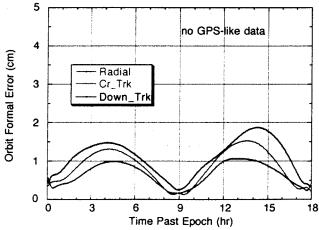


Fig. 4. VSOP-2 orbit determination error with conventional GPS data and accelerometry

Even without GPS-like measurements, the orbit formal error is well below 1 cm in all 3 components near the perigee. At higher altitudes, however, the orbit error grows to about 2 cm due to the lack of GPS measurements at these altitudes. The higher errors near the second occurrence of apogee (13.5 hr) are due to even fewer GPS measurements as compared to the first (cf. Fig. 3).

#### **GPS-like Measurements**

GPS-like measurements above 20,000-km altitude reduce the data scarcity and strengthen the observing geometry. Fig. 5 shows the location of ground sites to be assumed for receiving GPS-like measurements. The first 6 sites are fairly uniformly distributed. The additional 6 sites are selected to be in the northern hemisphere where VSOP-2 apogees occur. Fig. 6 shows the elevation angles of the GPS-like measurements when a global network of 6 and 12 sites are used. With a 6-site network, only 2 are usable near the first apogee occurrence and only 1 is usable near the second apogee occurrence. With a 12-site network, the number of usable sites increases to 4 and 6, respectively.

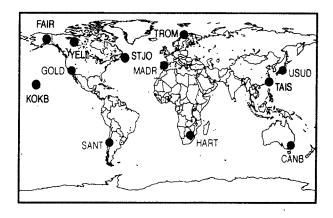


Fig. 5 Ground sites for GPS-like measurements

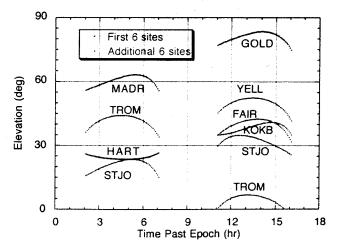


Fig. 6. Elevation angles of GPS-like measurements

Adding GPS-like data from 6 ground stations when VSOP-2 altitude is higher than 20,000 km reduces VSOP-2 altitude formal error, as shown in Fig. 7. Near the first apogee occurrence, only two stations (HART and MADR) are involved; the orbit near apogee is improved to about 1 cm. Near the second apogee occurrence, only one station (GOLD) is involved; the orbit near apogee is improved to better than 1.5 cm.

Adding GPS-like data from all 12 ground stations when VSOP-2 altitude is higher than 20,000 km reduces the orbit component formal errors to below 1.2 cm at all altitudes, as shown in Fig. 7.

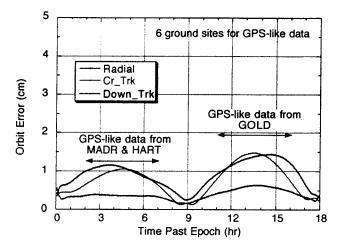


Fig. 7. VSOP-2 orbit determination error with conventional GPS data, accelerometry and GPS-like data from 6 ground sites

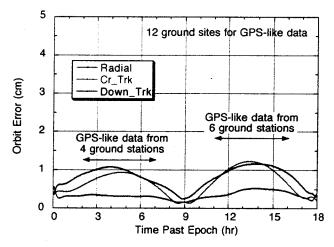


Fig. 8. VSOP-2 orbit determination error with conventional GPS data, accelerometry and GPS-like data from 12 ground sites

## **GPS Orbit and Clock Errors**

The effects of GPS orbit and clock errors, as simulated by differencing two GPS solutions with an RMS difference

of ~20 cm, for the case with GPS-like measurements from 12 sites are assessed. These are then scaled down by a factor of 4 to reflect the expected 5-cm GPS orbit and clock errors. The results are shown in Fig. 9 where the effects on VSOP-2 orbit determination are below 1 cm during the first rev and below 2 cm during the second rev.

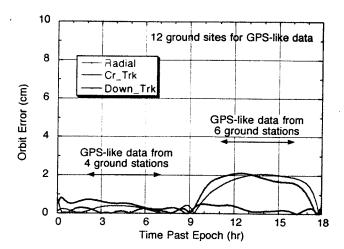


Fig. 9. Expected effects of GPS orbit and clock errors on VSOP-2 orbit determination with data as in Fig. 8

The results in Fig. 9 have been obtained with the data weighted according their assumed data noise in Table 1. The carrier phase data, which were degraded by GPS orbit and clock errors in the same way as the pseudorange data, became overweighted as compared to the pseudorange data. This has resulted in higher sensitivity to GPS orbit and clock errors. Such overweighting of carrier phase data will not happen when GPS orbits and clocks are simultaneously estimated with ground-based GPS measurements in actual data filtering process.

# **Ultra-Precise Accelerometry**

The above analysis has based on the assumption for the accelerometry to be of 1 nm/sec<sup>2</sup> accuracy as shown in Table 1. With this level of accelerometry quality, VSOP-2 orbit can be determined to an accuracy of about 2 cm. To further improve the orbit accuracy to the 1-cm level. accelerometry of higher quality is investigated. Ultraprecise accelerometers at the level of 0.1 nm/sec2 is now available and will be carried onboard the two spacecraft of Gravity Recovery and Climate Experiment (GRACE) mission [6]. When such ultra-precise accelerometers are used, VSOP-2 orbit improves to about 1.5 cm, limited by the GPS orbit errors, as shown in Figs. 10 and 11. Here the assumptions have been the same as for Figs. 8 and 9 except for the 0.1 nm/sec2 accelerometry. To assure 1-cm accuracy at all times, better determination of GPS orbits and clocks will be required.

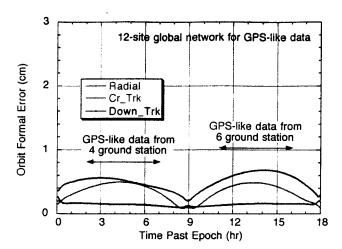


Fig. 10. VSOP-2 orbit determination error with conventional GPS data, ultra-precise accelerometry and GPS-like data from 12 ground sites

## **CONCLUSIONS**

A covariance/simulation analysis has been carried out to assess the potential VSOP-2 orbit determination accuracy. The results of this analysis show that a combination of conventional GPS measurements and onboard accelerometry is capable of determining the orbit to well below 1 cm in all three components near the perigee. At higher altitudes where GPS measurements are scarce, the orbit error can still be maintained at the centimeter level by adding GPS-like measurements.

In the covariance/simulation analysis, only the data noise and GPS orbit and clock errors have been included as the error sources. Station location and earth orientation parameter errors are expected to be maintained at the millimeter level with continuous VLBI and worldwide GPS measurements. Tropospheric delays can be accurately calibrated by state-of-the-art water vapor radiometers and/or accurately estimated by the abundance of GPS measurements. With the inclusion of these errors, the VSOP-2 orbit determination accuracy is expected to slightly degrade, but will remain at the centimeter level.

# **ACKNOWLEDGMENT**

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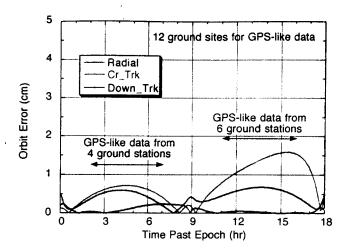


Fig. 11. Expected effects of GPS orbit and clock errors on VSOP-2 orbit determination with data as in Fig. 10

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